

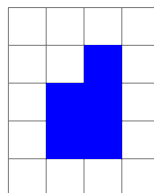
## Tiling the Plane

This is a “paper and pencil” activity where you use different colored pens or markers to fill in the squares of  $32 \times 32$  grid. The general algorithm we will be doing is called *tiling* and you see examples of this every day. Figure 1 shows a tiled floor.



Figure 1: A tiled floor.

In general, tiling consisting of placing one or more shapes having one or more colors on a surface, such as a floor. There are lots of different shapes that we can use when tiling. In this challenge, we will be doing tiling using the following shape shown below which belongs to the category of tiles called *pentominoes*. A pentomino is a tile constructed from five unit squares (*penta*, whose origin is the Greek word *pente*, means five). Using 5 unit squares it is possible to construct 12 distinct pentomino tile shapes.



Imagine you have a bunch of *tiles* having this shape. Also imagine the tiles you have come in different colors. You could then fit together these tiles like a jigsaw puzzle shown in Figure 2. For example, you could tile the floor of your house using these tiles.

Note that in the pattern shown below pieces that touch each other should have different colors and that pentomino shapes are rotated and reflected. What is the fewest number of colors needed to accomplish this tessellation? Do other tessellation patterns exist? Do patterns exist that require no rotation or reflection of the pentomino?

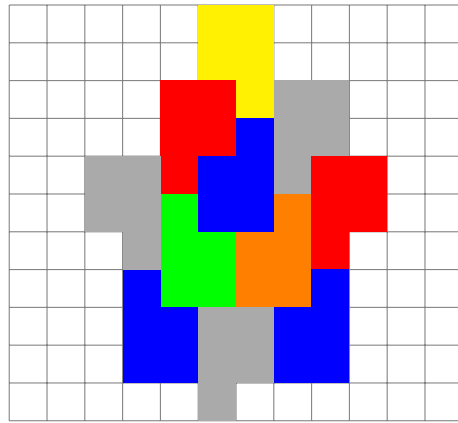


Figure 2: A tessellation using both rotation and reflection of the given pentomino.