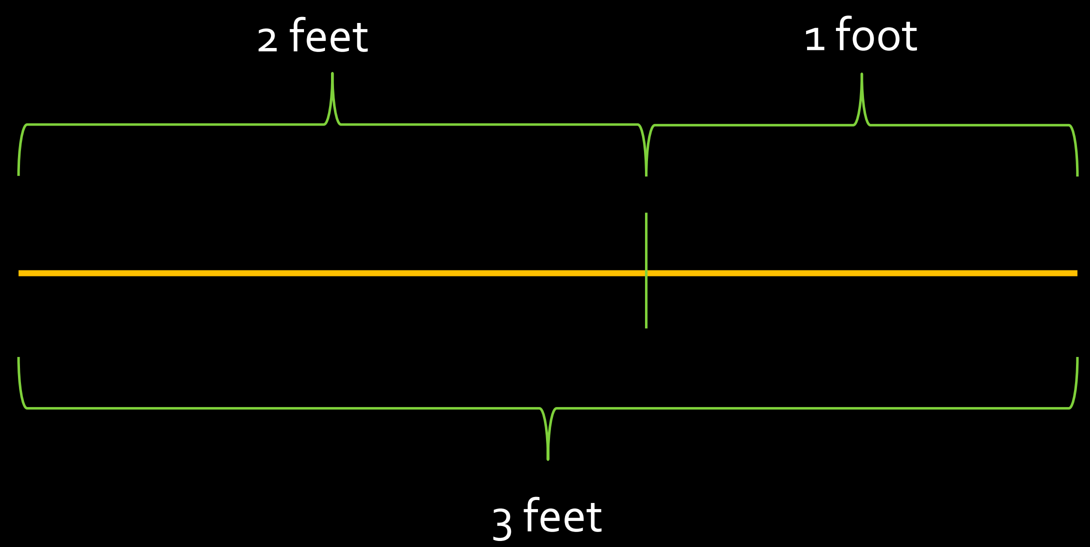
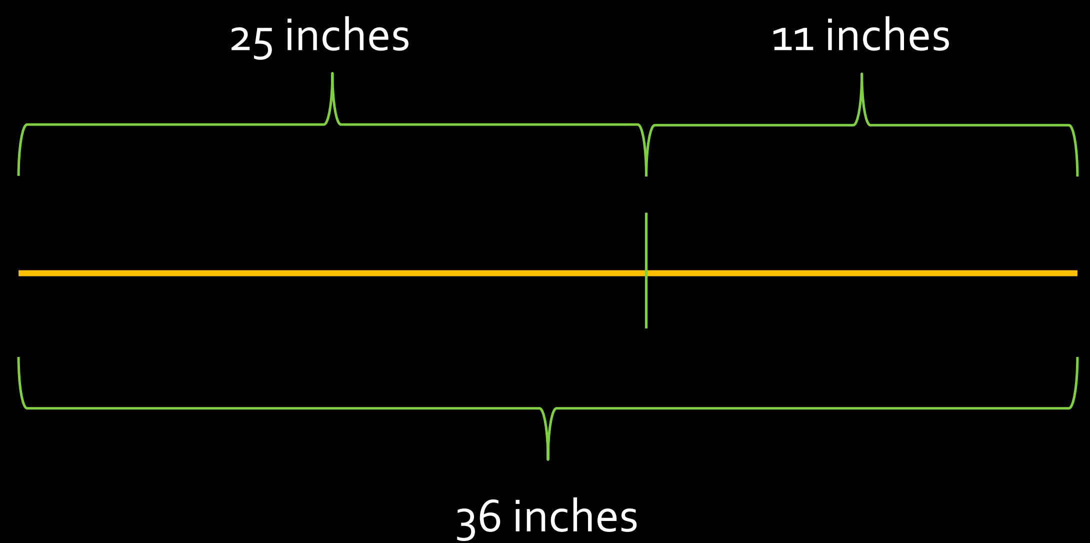
THE GOLDEN RATIO AND THE FIBONACCI NUMBERS

Common Measures



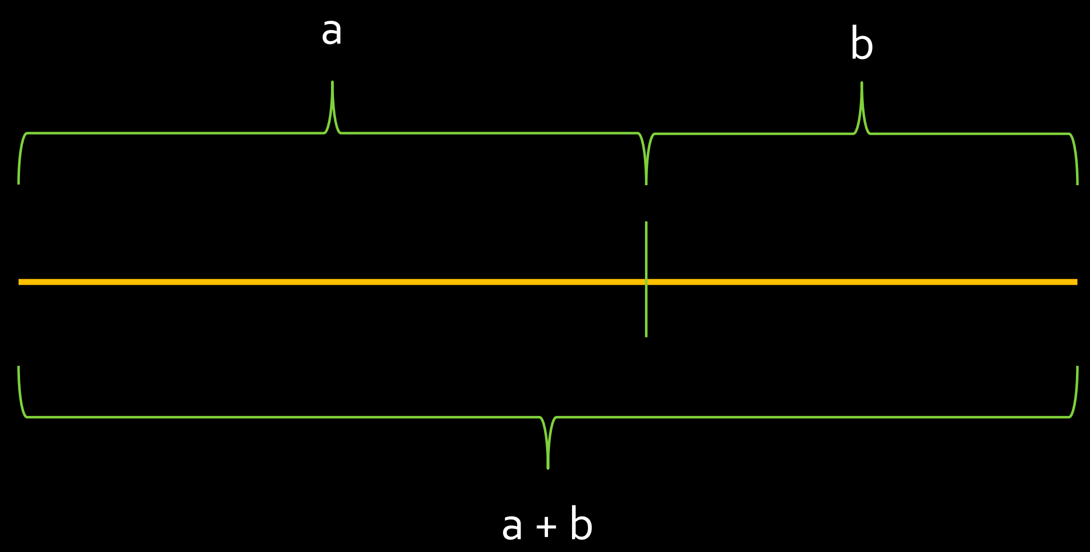
$$\text{Ratio} = \frac{3}{2} \qquad \text{Ratio} = \frac{2}{1}$$

Common measure = 1 foot



Ratio $= \dfrac{36}{25}$ Ratio $= \dfrac{25}{11}$

Common measure = 1 inch



$$\frac{a+b}{a} = \frac{a}{b}$$

Incommensurable!

(no fraction of a foot can be used to measure this distance)

φ

1.6180339887498948...

The origins of φ are shrouded in the mists of time

The Golden Ratio: φ

- In modern times is denoted by the symbol phi: φ
- Known to Euclid (300 B.C.) as a result of solving:

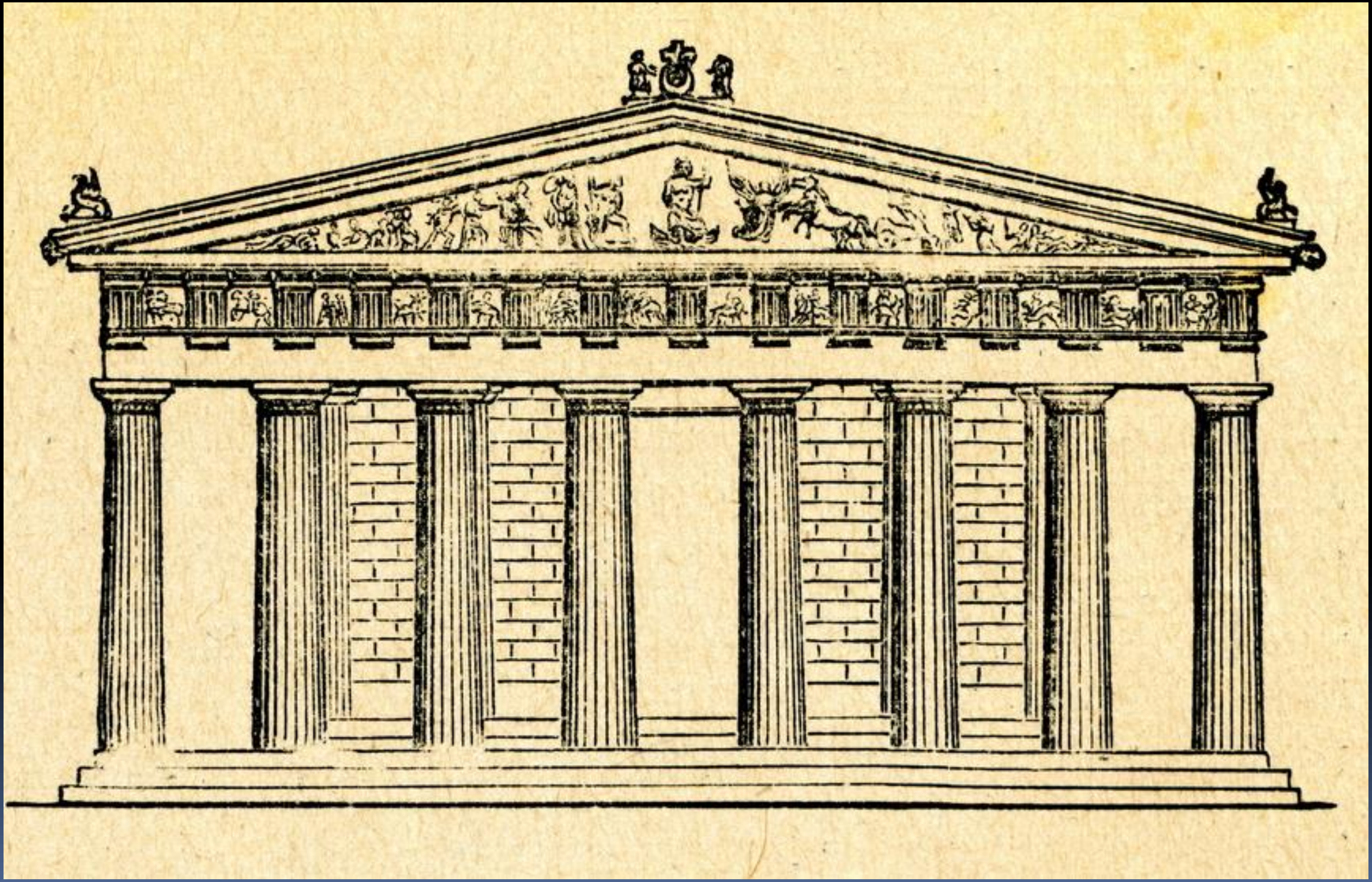
$$x^2 - x - 1 = 0$$

- A number of painters and architects have used the golden ratio in their work
- The length of a diagonal of a regular pentagram, whose sides have unit length, is φ
- Occurs in nature – represents a **growth pattern**

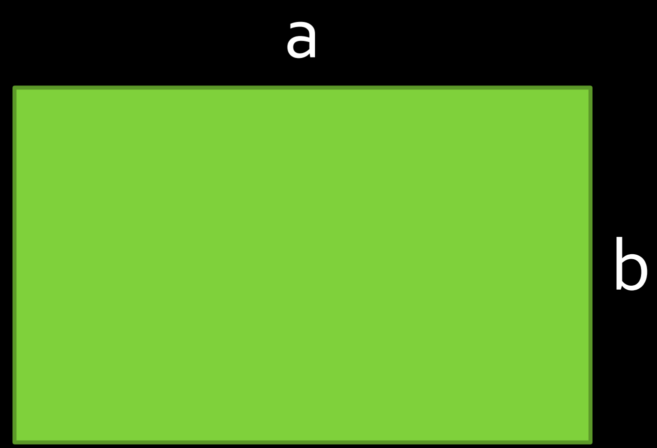
Legend and Speculation

- Was known to the ancient Egyptians.
- Was used to form the dimensions of the Great Pyramids of Egypt.
- Was applied to the design of the Parthenon.
- Was used in the design of Notre Dame in Paris.
- Was used in the construction of the Taj Mahal.

The Parthenon

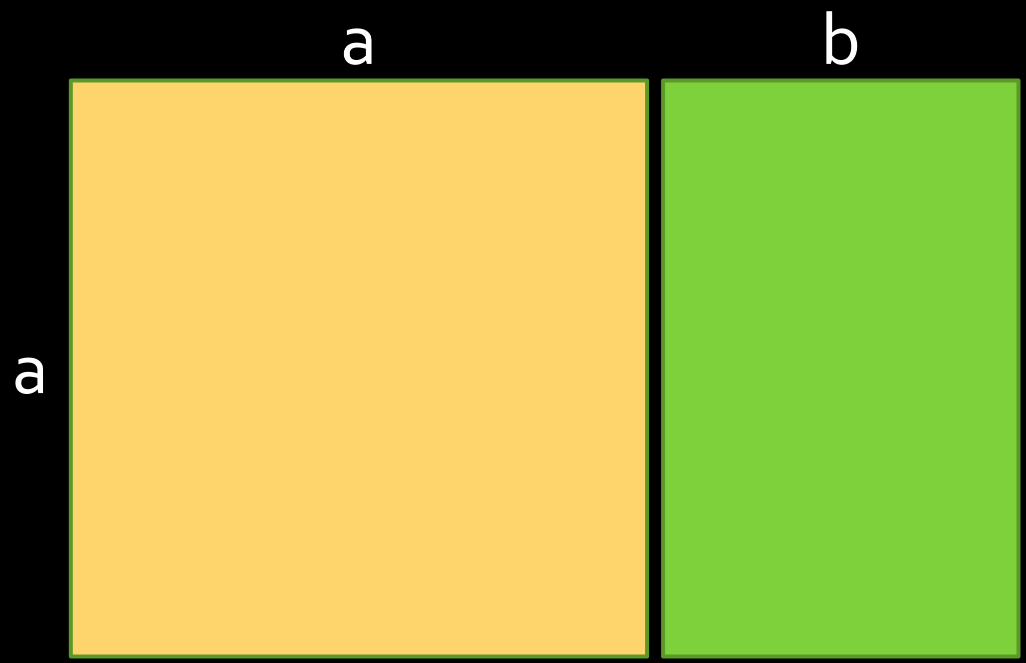


Ratio of a Rectangle



Ratio = $\frac{a}{b}$

Another Ratio



Ratio = $(a+b)/a$

These two rectangles have a **divine proportion** if:

$$\frac{a+b}{a} = \frac{a}{b}$$

The Algebra

$$\frac{a+b}{a} = \frac{a}{b}$$

$$b(a+b) = a^2$$

$$a^2 - ab - b^2 = 0$$

Letting $b = 1$ gives us:

$$a^2 - a - 1 = 0$$

Whose only positive solution is φ

Golden Ratio

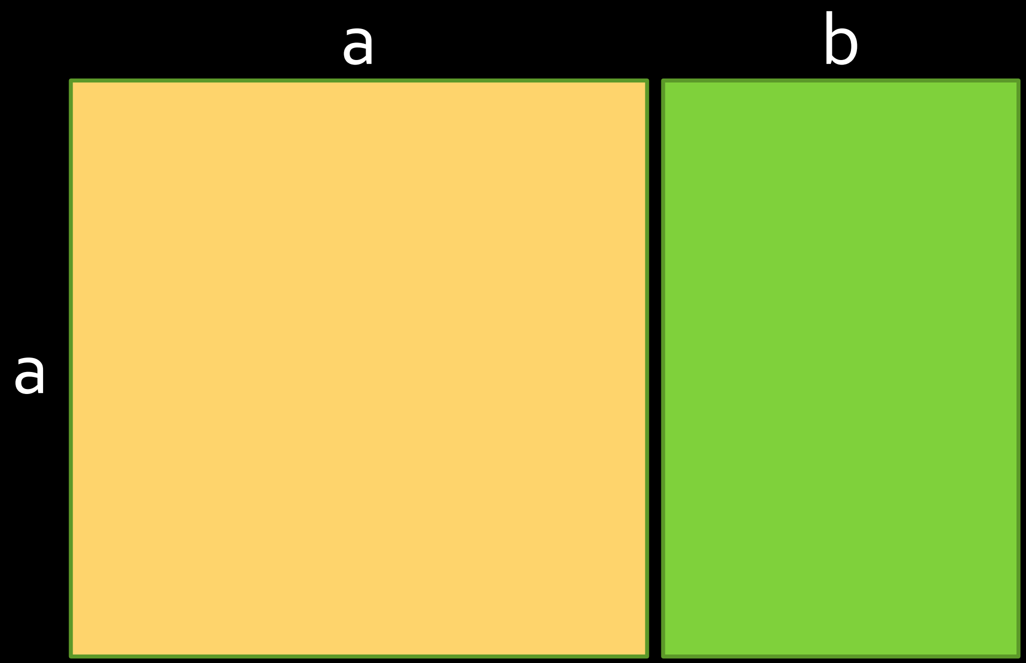
- Numerically the golden ratio is:

$$\varphi = \frac{1+\sqrt{5}}{2} = 1.61803\ \ldots$$

- This comes from solving $x^2 - x - 1 = 0$ using the quadratic formula:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- All rectangle pairs that are in divine proportion to each other will have this ratio.



Ratio = $(a + b)/a$

If these two rectangles have a divine proportion then:

$$\frac{a+b}{a} = \frac{a}{b} = \varphi$$

Fibonacci Numbers

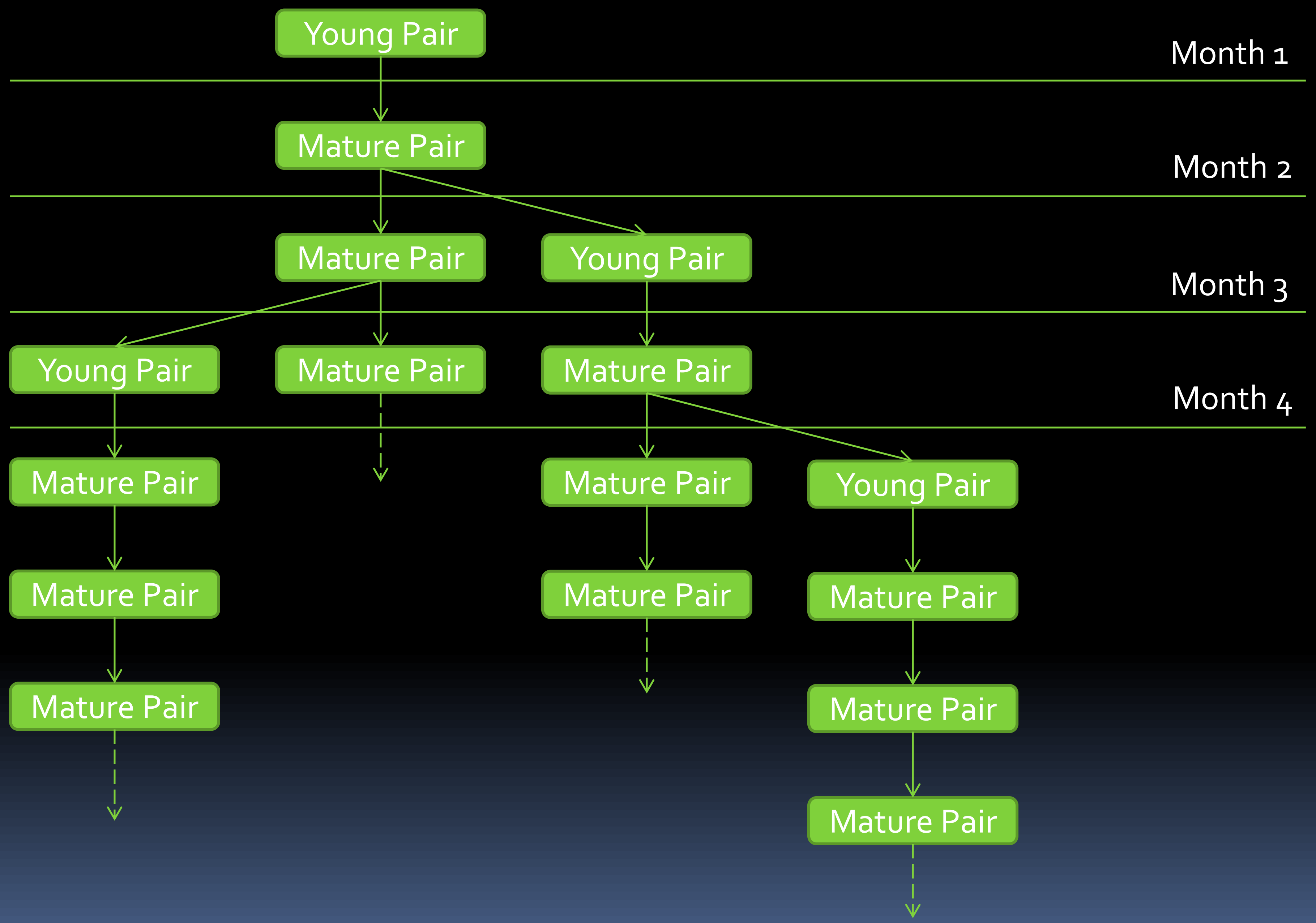
The Original Problem

- Stated by Fibonacci (whose original name was Leonardo of Pisa) in the year 1202
- Gives a recursive rule for computing the total number of rabbit pairs under “ideal” reproductive circumstances.

Problem Statement

- Start with a **rabbit pool** containing one pair of newly born rabbits (one male and one female)
- A newly born rabbit takes one month to reach **reproductive maturity**
- The **gestation period** of a reproductively mature female rabbit is one month
- A female rabbit will always give **birth** to two rabbits – one male and one female
  - This newly born pair is **added** to the rabbit pool

- Question: **How big** is the rabbit pool after
  - 12 months?
  - n months?


Young Pair
Month 1
Mature Pair
Month 2
Mature Pair
Young Pair
Month 3
Young Pair
Mature Pair
Mature Pair
Month 4
Mature Pair
Mature Pair
Young Pair
Mature Pair
Mature Pair
Mature Pair
Mature Pair
Mature Pair
Mature Pair

Fibonacci Sequence

- Starting from 1

  1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, …

- Starting from 0

  0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, …

In Flowers

The pedal count of many flowers are Fibonacci numbers (this is a known growing pattern)

1, 2, 3, 5, 8, 13, 21, 34, 55, 89,…

white calla lily

1, **2**, 3, 5, 8, 13, 21, 34, 55, 89,…

Euphorbia

1, 2, **3**, 5, 8, 13, 21, 34, 55, 89, …

Trillium

1, 2, 3, **5**, 8, 13, 21, 34, 55, 89, …

Buttercup

1, 2, 3, 5, **8**, 13, 21, 34, 55, 89, …

Bloodroot

1, 2, 3, 5, 8, **13**, 21, 34, 55, 89, …

Black eyed Susan

1, 2, 3, 5, 8, 13, **21**, 34, 55, 89, …

Shasta Daisy

1, 2, 3, 5, 8, 13, 21, **34**, 55, 89, …

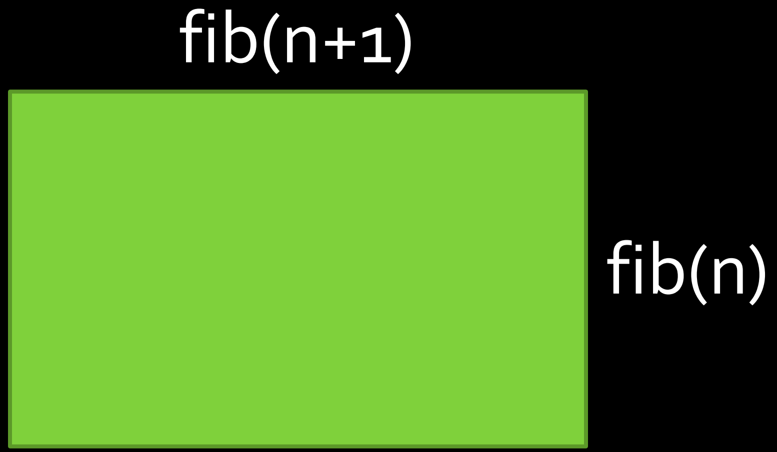
Field Daisies

1, 2, 3, 5, 8, 13, 21, 34, **55**, **89**, …

Michelmas Daisies

$$\lim_{n \to \infty} \frac{fib(n+1)}{fib(n)} = \varphi$$

Approaching the Golden Ratio



1, 1, 2, 3, 5, 8, 13, 21, 34, ...