A Gallery of Circles and Rings

bricklayer.org

Contents

1 Art Works

In Bricklayer, a circle can be created in coding Levels 2 and 3 using the function *circleXZ*. When creating a circle in Bricklayer, it is possible to control its size, color, and position through the use of parameters. A YouTube video describing the *circleXZ* function, its mathematical foundations (e.g., radius, diameter, and center), and a demonstration of how it can be used within a bricklayer-lite program can be found at `circles.bricklayer.org`.

In Bricklayer, a ring can be created in coding Levels 2 and 3 using the function *ringXZ*. When creating a ring in Bricklayer, it is possible to control its size, thickness, color, and position through the use of parameters. A YouTube video playlist describing the *ringXZ* function, its mathematical foundations (e.g., radius, thickness, diameter, and center), and a demonstration of how it can be used within a bricklayer-lite program can be found at `rings.bricklayer.org`.

1.1 Circle Sense

Picture 1: Circle Sense by Victor Winter and Betty Love

Create a work of art, like the one shown in Picture 1, containing circles having various sizes, colors, and positions. When imagining and creating your work of art, you should think about the following:

- What is the purpose of creating a particular distribution of size, color, and position?
- What would be the aesthetic effect of a random distribution?
- How would you (personally) go about deciding whether a collection of circles conforms to a: (1) a random color scheme, (2) a random size scheme, and (3) a random positioning scheme?
- How is a uniform distribution of sizes, colors, and positions different from a random selection?
- Can attributes be selected in such a way so that their composition has a particular aesthetic feel? For example, how would one go about creating a work of art that is similar to the drops of paint that you might find on the floor or table of an art studio?

The process used to create the artifact shown in Picture 1 is typically evolutionary in nature, and decisions involving the addition of circles are the result of a visual feedback loop that governs the suggestion of a circle's size, color, and shape. For example, an artifact may, in the mind's eye, look "out of balance" or incomplete. In such cases, the addition of a circle having a particular size, color, and position would improve the artifacts appearance. To add such an imagined circle requires an understanding of (1) how the radius of a circle relates to its size, and (2) how the relationship of the center of this circle relates to its neighboring circles (e.g., the new circle is to the left of the blue circle and slightly above the yellow circle, etc.). The proper sizing and placement of the circle can be obtained iteratively through a trial and error process. However, a clearer understanding of the underlying mathematics allows circles to be sized and placed more rapidly allowing for a smoother, quicker, and more engaging feedback loop.

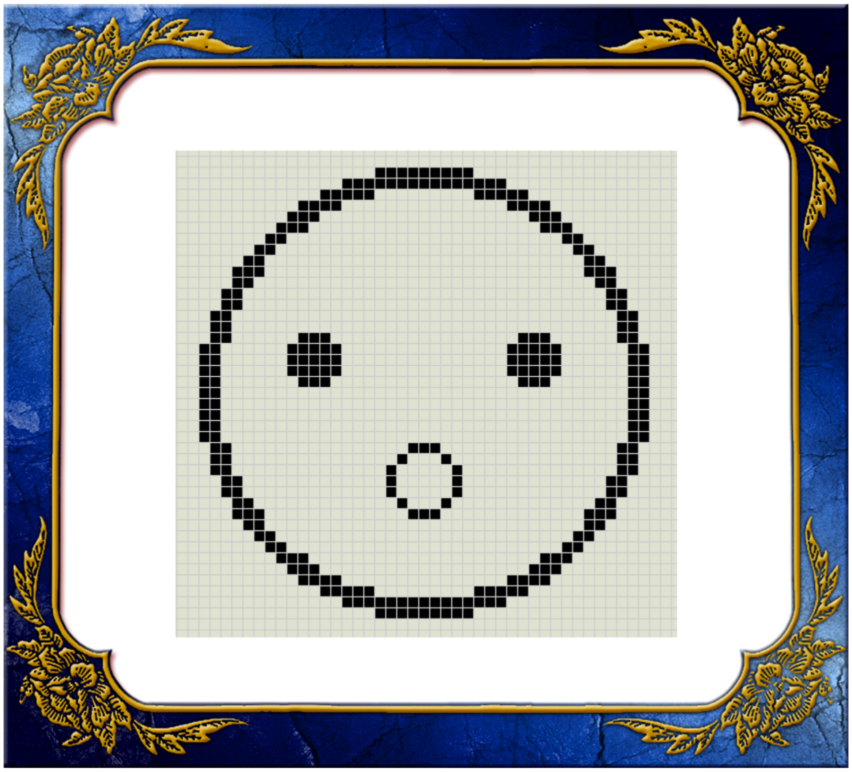
Math Standards

MA [4-8].3.2	Coordinate Geometry: Students will determine location, orientation, and relationships on the coordinate plane.
MA 8.3.2.a	Perform and describe positions and orientation of shapes under single transformations including ... translations, ... and dilations on and off the coordinate plane.

Art Standards

FA [2,5,8,12].2.1.b	unique student interpretation, imagination, styles, themes, subjects, personal voice, intention
FA [2,5,8,12].2.1.c	engage senses and emotions, aesthetics
FA [2,5,8,12].2.1.d	color, shape, pattern, symmetrical, asymmetrical, emphasis, multiple solutions
FA [2,5,8,12].2.1.e	technique, skill, craftsmanship (assumes coding and math techniques qualify)
FA [2,5,8,12].2.2.b	display, presentation skills, analyze, create a portfolio
FA [2,5,8,12].2.2.c	presentation venue and mode, purpose
FA [2,5,8,12].2.3.a	color, mood, style, individuality
FA [2,5,8,12].2.3.b	use of pattern, symmetry, shape, balance, themes, styles, principles
FA [2,5,8,12].2.3.c	interpretation, mood, feeling, message, compare and contrast
FA [2,5,8,12].2.3.d	personal interpretation, message, effects of display, critique
FA [2,5,8,12].2.4.c	purpose, function, form, aesthetic theory
FA [2,5,8,12].2.4.d	story, familiar experience, connection to world

1.2 The Face of Awe

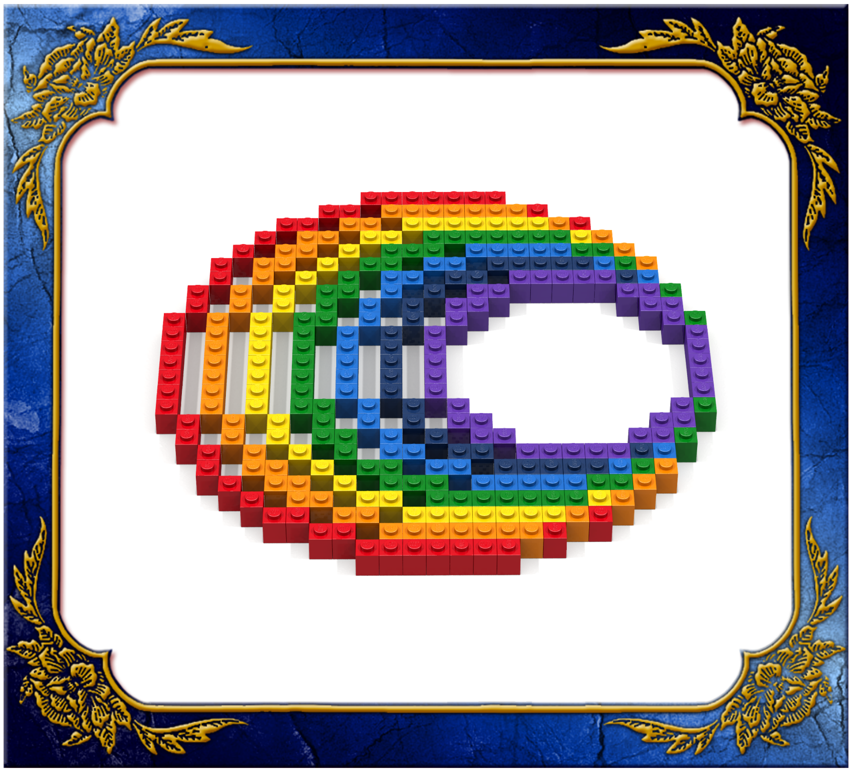


Picture 2: The Face of Awe by Kate Sherwin

Cultural Significance. Emoji are ideograms used in electronic messages and other forms of electronic media. Emoji, a composition of the Japanese words for “picture” and “character”, originated in Japan in 1999 for cell phone use. The original set of emoji was developed under the supervision of Shigetaka Kurita and contains 176 12x12 pixel images. This original set, which has been recognized as forming the beginning of a new visual language, can be found in the Museum of Modern Art.

Write a Bricklayer program that creates an emoji-like artifact similar to the one shown in Picture 2. Things to consider when creating your artifact are the effects of proportionality and symmetry. For example, how would the mood/emotion that is conveyed by the emoji be affected if the eyes were moved up, down, to the left, to the right, closer together, or farther apart? How is the mood affected by the size of the mouth and eyes? Is there a connection between the size of the eyes and the size of the mouth?

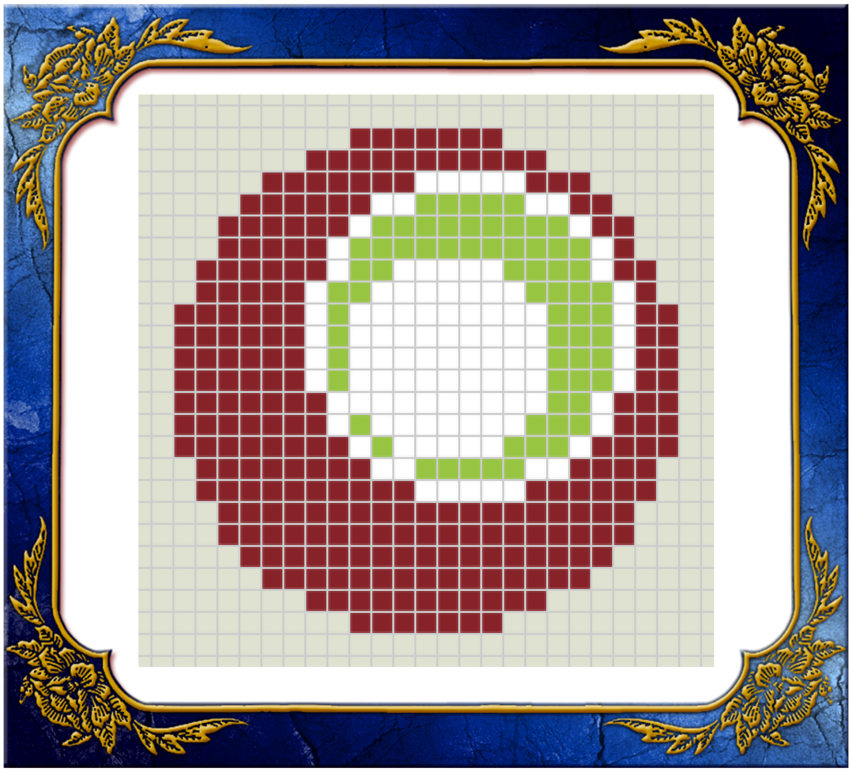
1.3 Migrating Rings



Picture 3: Migrating Rings by Victor Winter, Jennifer Winter and Betty Love

Write a Bricklayer program that creates an artifact similar to the one shown in Picture 3. Notice that in this artifact, rings overlap on their right sides. Also notice that, on their left sides, rings are separated from one another by a distance of 1 cell. This “migrating ring” effect can be achieved by moving ring centers along a horizontal axis and appropriately increasing ring sizes. The rest is left for you to figure out.

1.4 Through the Sphere



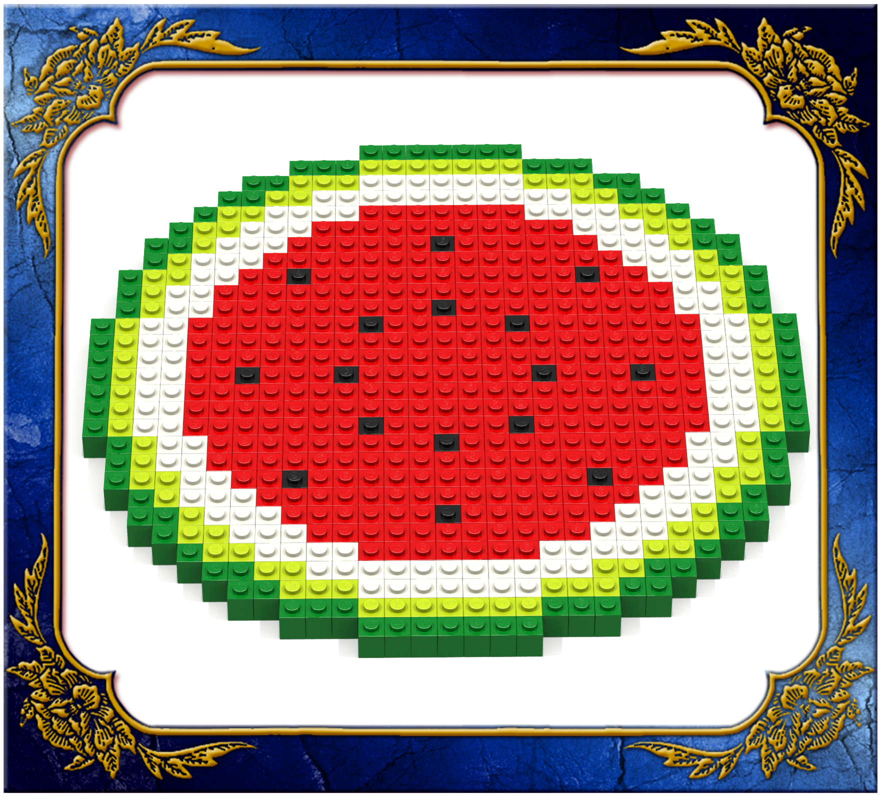
Picture 4: Through the Sphere by Kate Sherwin

Write a Bricklayer program that creates an artifact similar to the one shown in Figure 4.

Perspective and shading are two of the primary techniques used to give a three dimensional appearance to a two dimensional work of art. In the piece shown in Picture 4, circles are composed to give the viewer the impression that the artifact is spherical. In particular, migrating circles of white and light green are used to give the the impression of reflection and perspective.

It is widely recognized that words influence thought. In this context, the title of a work of art can influence the perceptions of the viewer. With a different title, the viewer could interpret the artifact as the depiction of an olive or even a bead of jewelery.

1.5 Still Life on a Summer's Day



Picture 5: Still Life on a Summer's Day by Kate Sherwin

Cultural Significance. A *still life*, one of the principa; genres of Western art, is a depiction of inanimate objects such as natural food, flowers, drinking glasses, books, and so on. This style of painting dates back to ancient Greco-Roman art. However, its origins as an art genere stems from 16^{th} and 17^{th} century paintings from the Netherlands.

Write a Bricklayer program that creates an artifact similar to the one shown in Figure 5. Notice the symmetric positioning of the watermelon seeds. Suppose the seed on the lower right (the seed positioned at around 4 o'clock) was colored brown. Would the resulting artifact be *chiral*? In general, which seeds could be colored differently to produce chiral artifacts?

1.6 The Olympic Symbol

Picture 6: A Series of Olympic-ring artifacts with radius = 4, 5, 6 by Kate Sherwin and Victor Winter

Cultural Significance. The official symbol of the Olympics (since 1915) consists of five interlocking rings. The rings represent the five inhabited parts of the world, and the design was created by Pierre de Fredy, Baron de Coubertin.

Write one or more Bricklayer programs that create artifacts, similar to the artifacts shown in Picture 6. Notice the rings in these Olympic-ring artifacts are overlapping but not interlocking.

> **Tip.** When performing the mathematical reasoning related to the construction of an Olympic-ring artifact, it can be very beneficial to use the bricklayer-grid to sketch circle diameters as vertical/horizontal lines in order to reason about their absolute as well as relative positions.

When constructing an Olympic-ring artifact, the placement of rings within the artifact must satisfy the properties stated in Table 1. An interesting line of inquiry arises regarding the degree of "artistic license" possible within the set of artifacts that satisfy the properties of Table 1. This line of inquiry can be guided through questions of the following kind.

1. Will any artifact that satisfies the properties of Table 1 appear similar to the Olympic symbol?
2. Do the properties of Table 1 imply that the horizontal spacing between rings on the upper row is the same as the horizontal spacing between rings on the lower row?
3. Do you think the stated properties permit only one solution for a given radius? For example, will every program, regardless of programmer, produce an artifact having the same shape?

Property 1.	The artifact must contain 5 rings grouped into 2 rows. The upper row must contain 3 rings.
Property 2.	All rings must have the same radius.
Property 3.	Rings on the same row must be separated by at least one cell.
Property 4.	Rings on the same row must be evenly spaced.
Property 5.	Each ring on the lower row (i.e., the yellow and green rings) must overlap with exactly 2 rings on the upper row. In this context, two rings, A and B, are said to *overlap* if one or more colored cells from ring A fall within the empty center of ring B and vice versa.
Property 6.	The center of each ring on the lower row must lie on the midpoint of the centers of the 2 rings (in the upper row) that it overlaps.

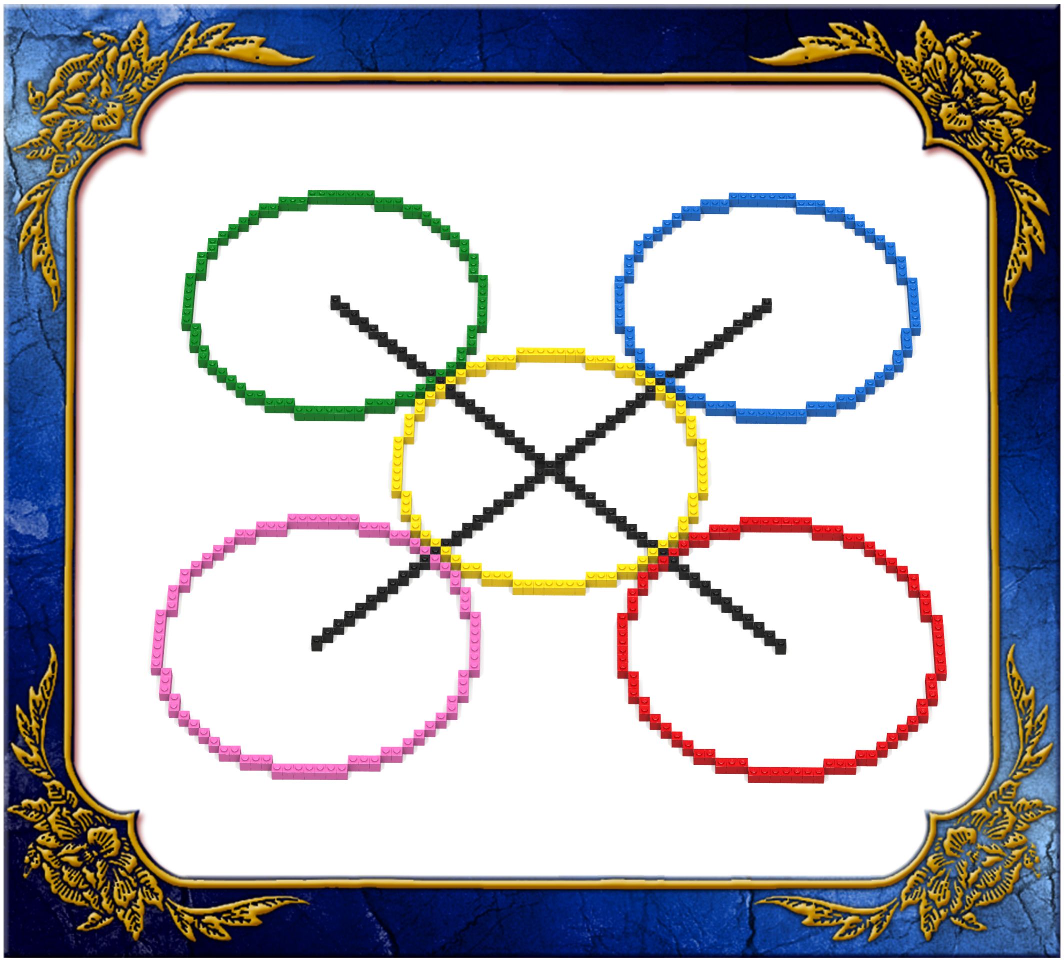
Table 1: Properties that a ring placement must satisfy to qualify as an Olympic ring artifact.

The first bricklayer-lite program should create an Olympic-ring artifact whose rings have radius 4. Mathematical reasoning can be used to determine the horizontal positioning of rings on a given row (e.g., $diameter + separation$ where $separation$ is an odd number). Mathematical reasoning can also be used to determine the positioning of the lower row of rings relative to the upper row (e.g., $center + (radius + 1) + separation\ div\ 2$). A trial-and-error approach can then be used to determine the vertical placement of the upper row relative to the lower row.

After completing the first program one can consider writing another program that creates an Olympic-ring artifact whose rings have a larger radius (e.g., radius = 5). The construction of this second program can be approached in one of two ways: one could start from the beginning or, alternately, one could try to modify the first program which created an Olympic-ring artifact whose rings had radius 4. The second approach will typically involve trial-and-error combined with mathematical reasoning. Ultimately, the issue is to determine the positions of all five ring centers as a function of the size of the radius – a value which is the same for all the rings. Specifically, how does a center of a particular ring shift in response to a change in the size of the radius. If this function can be articulated in code, then it is easy to create an Olympic-ring artifact whose rings have a variety of radiuses.

Observation. Note that when the radius of a ring is increased the size of the ring grows in all directions (e.g., up, down, to the left, and to the right). So, for example, if the first program positioned its Olympic-ring artifact on the edge of the build space (e.g., left and/or bottom) then an increase in the radius of the rings will cause portions of one or more of the rings to fall outside the build space. While this will not cause the program to crash, it will affect the appearance of the artifact produced.

1.7 Pythagorean Rings



Picture 7: Pythagorean Rings by Victor Winter and Betty Love

Write a Bricklayer program that creates an artifact, shown in Picture 7, in which five rings are placed adjacent to one another to form an X-like configuration with diagonal lines connecting the ring centers.

When constructing this artifact, the challenge is to determine the locations of the ring centers. This determination can be made through brute-force trial-and-error guided by some "circle sense". However, the labor involved in this approach increases as the rings grow in size. In the end, developing a precise mathematical understanding of the formulas governing the positioning of ring centers is a more effective way to solve this problem. It is this second construction approach that is discussed in the paragraphs that follow.

Note that, regardless of position and orientation, the distance between adjacent rings will always equal one diameter in length. For example, let c_1 denote a ring having radius r and center (x, z).

Let c_2 denote a ring having radius r that is adjacent-to and to-the-right-of c_1. In this case, the center of c_2 will be $(x + 2r + 1, z)$. Recall that, in Bricklayer, the diameter of a ring having radius r is $2r + 1$. (See the YouTube playlist `rings.bricklayer.org` for a discussion of why this is so.)

When creating the *Pythagorean Rings* artifact, we can begin by placing the first ring, which we will call c_1, at the location (r, r) where r is the radius of the ring. This will have the effect of positioning c_1 so that it is adjacent to the left side as well as the bottom of the build space. The primary challenge now is to figure out the location of the center of the middle ring, which we will call c_2. Since c_1 is adjacent to c_2, we know that the distance between their centers is one diameter d in length. From the diagonal lines and the artifact symmetry it can be deduced that the line segment connecting the centers of c_1 an c_2 forms the hypotenuse of an *isosceles right triangle* (i.e., a right triangle whose legs have equal length). Figure 8 summarizes the information at hand.

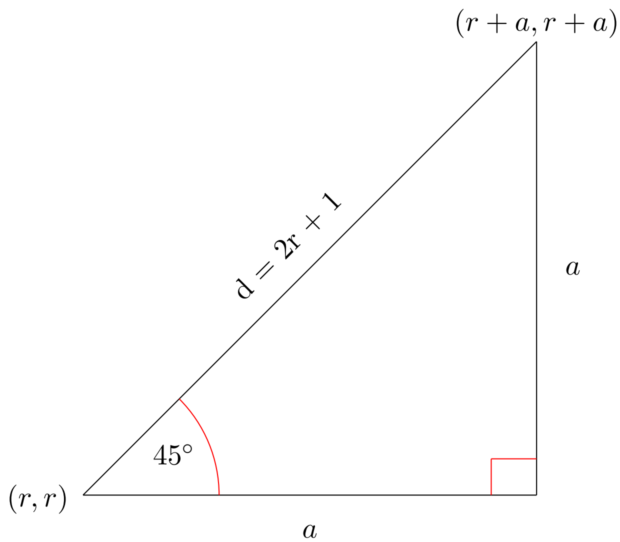


Figure 8: An isosceles right triangle with the lower left and upper right corners denoting the centers of c_1 and c_2 respectively.

In order to determine the center of c_2, one must determine the length of the leg of the isosceles right triangle. Once the center for c_2 has been determined, reasoning based on symmetry can be used to determine the centers of the remaining rings in the artifact.

center	**position**	**description**
c_1	(r, r)	lower-left ring (having radius r)
c_2	$(r + a, r + a)$	middle ring
c_3	$(r + 2a, r + 2a)$	upper-right ring
c_4	$(r, r + 2a)$	upper-left ring
c_5	$(r + 2a, r)$	lower-right ring

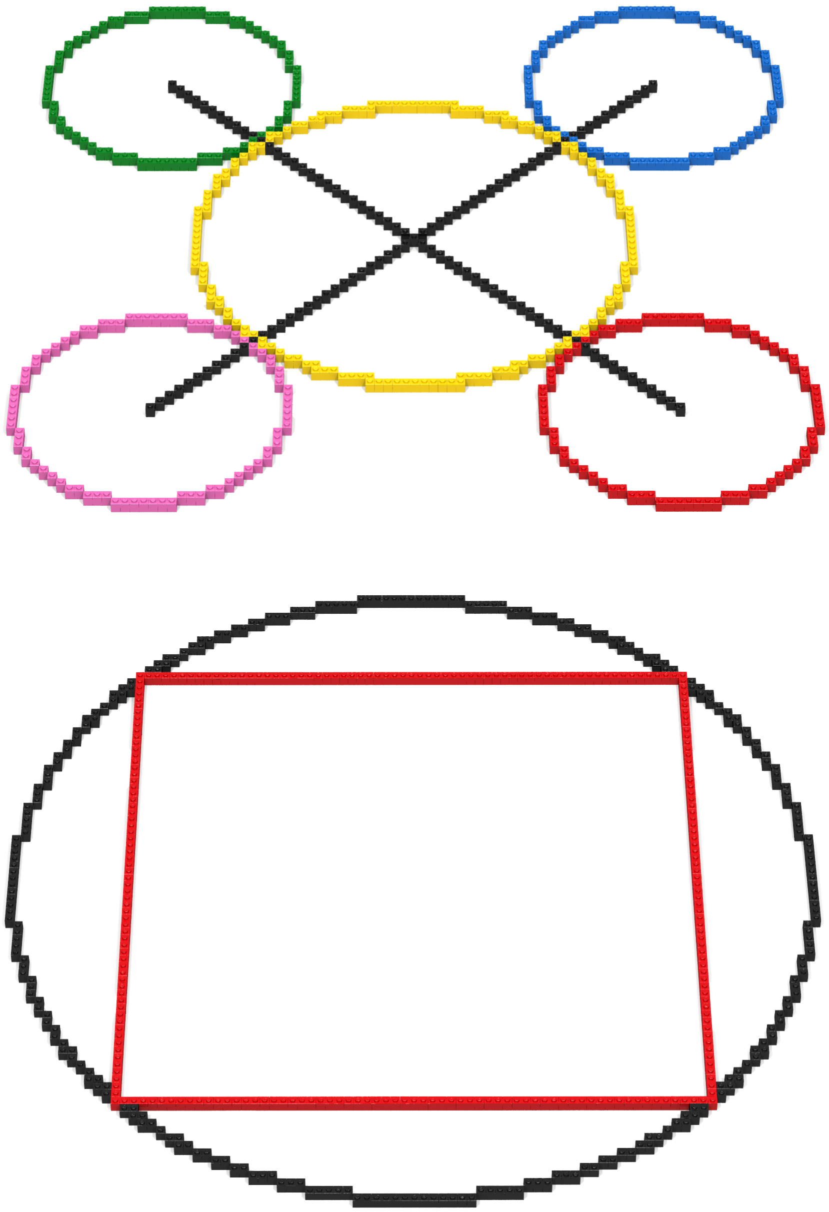
Table 2 shows the algebraic steps that solve for the leg (a) of the isosceles right triangle. This lets us conclude that the center of c_2 can be caclulated for any radius r using the following formula.

$$c_2 = (r + \sqrt{2}(r + \frac{1}{2}), r + \sqrt{2}(r + \frac{1}{2})) \tag{1}$$

$a^2 + b^2 = d^2$	Pythagorean Theorem
$a = b$	Because the right triangle is an isosceles right triangle
$a^2 + a^2 = d^2$	Substituting for b
$2a^2 = d^2$	Basic math
$a^2 = \frac{d^2}{2}$	Dividing both sides by 2
$a = \sqrt{\frac{d^2}{2}}$	Taking the square root of both sides
$a = \frac{d}{\sqrt{2}}$	Simplifying
$a = \frac{d}{\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}}$	Multiplying the right-hand side by a "well chosen 1"
$a = \frac{d}{2} * \sqrt{2}$	Simplifying
$a = \frac{2r+1}{2} * \sqrt{2}$	Substituting for d
$a = \sqrt{2}(r + \frac{1}{2})$	Simplifying

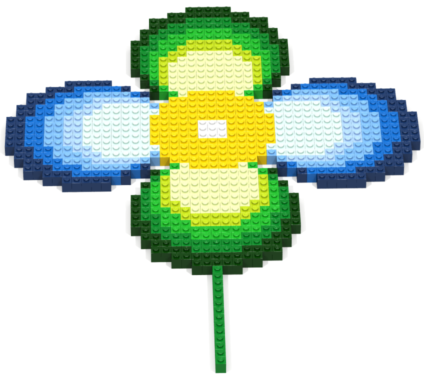
Table 2: Calculating the center of the middle ring c_2.

Ideas based on the Pythagorean theorem forms the foundation for the construction of a variety of artifacts. Two such artifacts are shown in Figure 9.

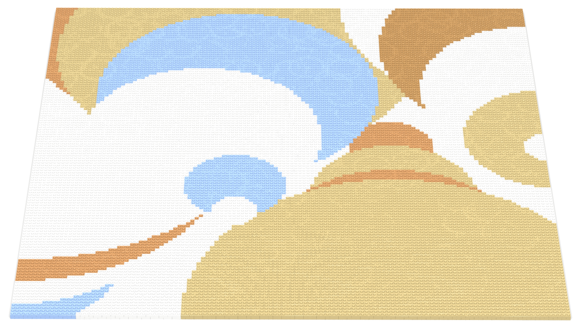


Picture 9: Pythagorean artifacts

2 Various Works by Artist-Programmer-Mathematicians

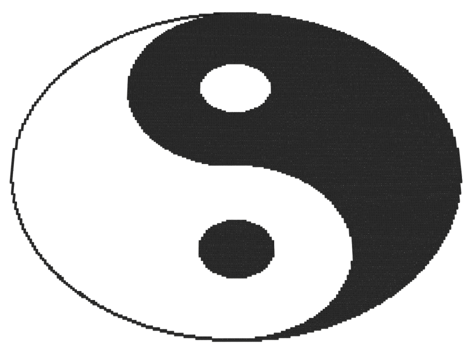


(a) By Danielle Meadows



(b) By Nicholas Dolezal

(c) By Heilo Ribeiro da Cruz



(d) By Victor Winter